A REAL-TIME 3D VISUALIZATION APPROACH FOR THE APPEARANCE OF CROP LEAVES

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Abstract

A real-time 3D modeling and visualization approach for simulating the appearance of crop leaves are presented. The basis of this method is an appearance model in terms of bidirectional reflectance distribution functions (BRDF) and bidirectional transmittance distribution functions (BTDF) whose diffuse and transmittance term are derived using some physiological parameters. Cook - Torrance is employed to compute the glossy reflectance in our model. For simulating the spatial-varying optical properties of leaves, four appearance textures are utilized to control the parameters of appearance model. In order to achieve a WYSIWYG display result, we simplify the light computing by decomposing light environment into a directional light source and some environment light sources. The experimental results demonstrate that the proposed approach is capable of generating different appearance of crop leaves by controlling the physiology parameters and achieving a visually satisfactory display result. Our method has a great potential to become an effective visualization tool for agricultural applications.

Introduction

3D modeling and visualization of crops can help workers understand the underlying crop traits and data much better than a simple table of numbers. Some applications of 3D technology in agriculture have presented this potential (Jeremy *et al.* 2004, Mebatsion *et al.* 2001, Lin *et al.* 2008, Falcão *et al.* 2006). 3D structure modeling technology has already been able to generate realistic and precise crop morphology (Lim and Honjo 2013, Quan *et al.* 2006, Qu *et al.* 2010, Guo *et al.* 2006, Ma *et al.* 2007). However, 3D modeling and visualization of the appearance of crop organs is still difficult due to the wide variations in material appearance and physiological structures. Because of the importance in the physiological function and morphology for crops, modeling of the appearance of crop leaves is an important and basic research work.

Some technologies have been proposed to generate realistic appearance of plants in computer graphics (Hanrahan *et al.* 1993, Wang *et al.* 2005, Franzke *et al.* 2003, Ralf *et al.* 2007). These methods derived specific appearance model for leaves with some optical parameters such as scattering and absorption coefficients which quantize the interaction between light and their intricate underlying structure and successfully obtained realistic results. While appearance models used in computer graphics are not suitable for agriculture and other related areas, because their parameters don't have typical agricultural or biological significance.

Only a few models utilized physiological parameters as parameters which may be employed for agricultural application possibly, such as ABM (algorithmic BDF model, BDF represents bidirectional surface-scattering distribution function) (Baranosk 1997), FSM (foliar scattering model) models (Baranosk 2001). But these two models spend several hours for generating an

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acceptable visualization result which would limit the usefulness of the models. Run time is another key factor to be considered in a visualization tool. Poor run time performance will make it not suitable to display massive data or give a dynamic visualization effect for decision support tools, eventually resulting in decline of the assist ability. We think that a WYSIWYG visualization tool must be flexible and robust.

Previously, we have presented a shading framework which has two physiological parameters : chlorophyll a + b content and carotenoids content (Miao *et al.* 2013), however this framework ignores many apparent details and can't guarantee real-time response with more than 20 thousand triangle faces. In this research, we present an enhanced appearance model with some physiological parameters and a real-time visualization method for simulation of the appearance of crop leaves which has an ability to become an effective visualization tool for agricultural application.

Materials and Methods

Appearance model: Our model describes leaf appearance in terms of a few parametric bidirectional reflectance distribution functions (BRDF) f_r and bidirectional transmittance distribution functions (BTDF) f_t . $f_r f_t$ as shown in Eqns. (1) and (2)(Wang *et al.* 2005)

$$f_r = \frac{1}{\pi} \rho_d + f_s \tag{1}$$

where ρ_d is the diffuse term that describes the diffuse reflection due to subsurface scattering and f_s is the glossy term describes the glossy reflection due to surface roughness. The BTDF has only a diffuse term because light transmitted through leaf tissue becomes diffuse and it can be written as

$$f_t = \frac{1}{\pi} \rho_t \tag{2}$$

We derive the diffuse term ρ_d and ρ_t based on a leaf radiative transfer model called "plate model" (Allen *et al.* 1967). The general formula of "plate model" (Jacquemoud *et al.* 1990) for reflectance ρ_{α} and transmittance τ_{α} can be written as follows:

$$\rho_{\alpha} = [1 - t_{av}(\alpha, n)] + \frac{t_{av}(90, n)t_{av}(\alpha, n)\theta^{2}[n^{2} - t_{av}(90, n)]}{n^{4} - \theta^{2}[n^{2} - t_{av}(90, n)]^{2}}$$
(3)
$$\tau_{\alpha} = \frac{t_{av}(90, n)t_{av}(\alpha, n)\theta n^{2}}{n^{4} - \theta^{2}[n^{2} - t_{av}(90, n)]^{2}}$$
(4)

where α is the maximum incidence solid angle, *n* is the refractive index, $t_{av}(a, n)$ is the transmissivity of a dielectric plane surface and can be exactly calculated, θ is transmission coefficient of the plate based on the absorption coefficient k:

$$\theta = (1-k)e^{-k} + k^2 \int_k^\infty x^{-1} e^{-x} dx$$
(5)

Jacquemoud (Jacquemoud et al. 1990) defined k as the following formula:

$$k = \frac{\sum K_i C_i}{N}$$

 K_i is the specific absorption coefficient relative to the leaf component *i* including chlorophyll a+b coefficient K_{ab} , carotenoids K_r , brown pigment K_b and dry matter absorption coefficients K_m ; C_i is the leaf content per unit leaf area of component *i* including chlorophyll a+b content C_{ab} (µg/cm²), carotenoids content C_r (µg/cm²), brown pigment content C_b (arbitrary units and dry matter content C_m (g/cm). N is the structure parameter relates to the cellular arrangement within the leaf which is used for identifying plant types.

Transmission coefficient θ is computed using the complex integral which would consume much computational time. In order to simplify the calculation, we used three piecewise functions to fit the original integral equation (5)

$$\begin{cases} \theta = 0.951e^{-1.463k} & k <= 1.0\\ \theta = 0.743e^{-1.25k} & 1 < k <= 3.0\\ \theta = 0.01 & k > 3 \end{cases}$$

Absorption coefficient K_i in the formula (5) are all spectrum values ranging from 400 to 2500 nm. Because the current display equipments are all RGB mode (R means red, G means green, B means blue), we have to convert K_i and n from 401 dimensional vectors into 3 dimensional RGB vectors. In this paper, we chose the value at 675 nm as R channel, 546nm as G channel and 435 nm as B channel, so the RGB chlorophyll absorption coefficient is 0.05914, 0.01221 and 0.0716, carotenoids is 0.0, 0.0073 and 0.168, pigment is 0.1572, 0.3568 and 0.4912, the dry matter is 2.3, 2.3 and 1.481. n is also a spectrum value, but in this paper, we make it equal to 1.42 according to the agreement in most computer graphic application.

Reflectance ρ_{α} is the Directional Hemispheric Reflectance Factor. We let α equals to 0, that makes the diffuse reflection maximum and glossy reflection minimum. In this paper, we assume that ρ_{α} equals to ρ_d to simplify the model. τ_{α} is the Directional Hemispheric Transmittance Factor, and the transmission of leaves is diffuse, so ρ_t equals to τ_{α} .

Through the above treatment, the only variable in (3) and (4) is θ . We substitute $\alpha = 0$ and n = 1.42 to (3) and (4), getting

$$\rho_{d} = \rho_{\alpha} = \frac{0.031 + 0.240\theta^{2}}{1.0 - 0.296\theta^{2}}$$

$$\rho_{t} = \tau_{\alpha} = \frac{0.44\theta}{1 - 0.001\theta^{2}}$$
(6)

The glossy term f_s describes the glossy reflection due to the rough surface of plant leaves. Bousquet established that the Cook - Torrance model can be used to describe the glossy optical properties of leaves (Bousquet *et al.* 2005). Cook-Torrance model is as follows:

$$f_{s} = \frac{F(n,\theta_{h})}{\pi} \frac{D_{B}(\theta_{h},\gamma)G(\theta_{i},\theta_{v},\theta_{h},\gamma)}{\cos\theta_{i}\cos\theta_{v}}$$
(7)

where

$$F(n,\theta_{h}) = 0.5 \left(\frac{g - \cos\theta_{h}}{g + \cos\theta_{h}}\right)^{2} \left[1 + \left(\frac{\cos\theta_{h}(g + \cos\theta_{h}) - 1}{\cos\theta_{h}(g - \cos\theta_{h}) + 1}\right)^{2}\right]$$
$$g = \sqrt{n^{2} + \cos^{2}\theta_{h} - 1}$$

$$G(\theta_{i}, \theta_{v}, \theta_{h}) = \min\left(1, \frac{2\cos\theta_{h}\cos\theta_{v}}{\theta_{h}}, \frac{2\cos\theta_{h}\cos\theta_{i}}{\theta_{h}}\right)$$
$$D_{B}(\theta_{h}, \gamma) = \frac{1}{\gamma^{2}\cos^{4}\theta_{h}}e^{-(\tan\theta_{h}/\gamma)^{2}}$$

where *n* is the refractive index. θ_i and θ_v are the illumination direction and viewing direction, respectively. θ_h is the half vector between θ_i and θ_v , called half angle. $F(n, \theta_h)$ is the Fresnel factor. $G(\theta_i, \theta_v, \theta_h)$ is the shadowing and masking term. $D_B(\theta_h, \gamma)$ is normal distribution term. γ is the roughness parameter and as γ decreases, $D_B(\theta_h, \gamma)$ become very high which makes the glossy reflection close to the specular reflection.

Substituting (6) and (7) into (1) and (2), the final appearance model becomes the following form and has 8 parameters, respectively C_{ab} , C_r , C_b , C_m , N, γ , θ_i , θ_v

$$f_r = \frac{1}{\pi} \frac{0.031 + 0.240\theta^2}{1.0 - 0.296\theta^2} + \frac{F(n, \theta_h)}{\pi} \frac{D_B(\theta_h, \gamma) G(\theta_i, \theta_v, \theta_h, \gamma)}{\cos \theta_i \cos \theta_v}$$
(8)
$$f_t = \frac{1}{\pi} \frac{0.44\theta}{1 - 0.001\theta^2}$$
where θ depends on $C_{ab}, C_r, C_b, C_m, N.$

Appearance texture: The reflectance properties of leaves is spatial-varying because of nonuniform distribution of physiological components. Texture image is an effective tool to simulate this feature and in our method four different textures were used including pigment texture, glossy texture and thickness texture. Appearance texture is a general designation for these four textures in this paper (Fig. 1).

Before generating appearance textures, we first make a RGB image called structure image (Fig. 1(a)) used to distinguish the main veins, secondary veins and mesophyll whose normalized pixel values in texture are, respectively (1,0,0), (0,1,0) and (0,0,1).

With the structure image, our method first generates a pigment texture. Pigment texture (Fig. 1(b)) is stored as a regular RGB image. The R, G, B values of pixels respectively represents the ratio relationship of C_{ab} , C_r , and C_b at different point represented by the notations V_a , V_c , V_b . We observed that the color of mesophyll is more green and vein is more yellow and brown, so we set a greater value for V_a , a smaller for V_c and V_b of pixels belong to mesophyll than vein.

Glossy texture is stored as a gray images whose pixel values represent brightness ratio represented by the notations VB (Fig. 1(c)). We observed that main vein has the largest glossy intensity and mesophyll has the minimum intensity. So, we set a larger value for V_g for main vein pixels, and smaller for mesophyll pixels.

According to Beer-Lambert Law, the transmitting medium absorbs light relative to thickness. We use thickness texture (Fig. 1(d)) for the varying thickness of the leaf. This texture is also stored as gray images and the pixel V_t represents thickness ratio. We use a lager V_t for vein pixels and smaller V_t for mesophyll pixels.

Using appearance texture, the spatial-varying optical properties can be obtained by combining V_a , V_c , V_b , V_B and V_t with the appearance model (8) in the following form:

$$f_r = \frac{1}{\pi} \frac{0.031 + 0.240\theta^2}{1.0 - 0.296\theta^2} + V_B \frac{F(n, \theta_h)}{\pi} \frac{D_B(\theta_h, \gamma) G(\theta_i, \theta_\nu, \theta_h, \gamma)}{\cos \theta_i \cos \theta_\nu}$$
(9)

$$f_{t} = \frac{V_{t}}{\pi} \frac{0.44\theta}{1 - 0.001\theta^{2}}$$

$$\begin{cases} \theta = 0.951e^{-1.463k} & k <= 1.0 \\ \theta = 0.743e^{-1.25k} & 1 < k <= 3.0 \\ \theta = 0.01 & k > 3 \end{cases}$$

$$k = \frac{K_{ab}V_{a}C_{ab} + K_{r}V_{c}C_{r} + K_{b}V_{b}C_{b} + K_{m}C_{m}}{N}$$

Fig. 1. Appearance textures: (a) Structure image, (b) Pigment texture, (c) Glossy intensity and (d) thickness texture.

Lighting computation: In this section we present our real-time lighting algorithm to simulate the subtle interaction between plant leaves and light. We decompose the outdoor light environment into a directional light source (sun) and some environment light sources (sky).

We define the sun as a directional light source of direction L_i and irradiance E_s . Like most real-time rendering methods, the outgoing radiance $E_o(p)$ and transmitted radiance $E_t(p)$ at position p is computed by

$$E_o(p) = E_s(L_i \cdot N_r)M(p, L_i)f_r(p) = E_s \cos\theta_i fr M(p, L_i)f_r(p)$$

$$E_t(p) = E_s(L_i \cdot N_r)M(p, L_i)f_t(p) = E_s \cos\theta_i fr M(p, L_i)f_t(p)$$

where $f_r(p)$ and $f_t(p)$ are the BRDF and BTDF at position *p*. $M(p,L_i)$ is a light-visibility term represents that whether the leaf surface at position p is occluded from the direction Li , $M(p,L_i) = 1$ means no occlusion and 0 means occlusion. This term is computed using cascaded shadow maps technology (Zhang 2007). N_r is the normal vector at position p.

Sky light is the main outdoor environment light. In our method, we assume that the outgoing radiance lighting by sky is almost diffuse, then the radiance $E_e(p)$ can be computed by

$$E_{e}(p) = f_{r}^{d}(p) \int_{\Omega(N_{r})} E(\omega)(N_{r} \cdot \omega) M(P, \omega) d\omega$$

where $E(\omega)$ is the irradiance of sky light of direction ω . We calculated it based on accurate atmospheric scattering simulation model (Sean 2007). $\Omega(Nr)$ is the upper hemisphere of normal vector N_r . $M(p, \omega)$ is the light-visibility term. $f_r^d(p)$ is the diffuse term of the BRDF f_r at position p. A dynamic irradiance environment maps computing method is used for the above integral (Sean 2007) and screen-space ambient occlusion algorithm (McGuire 2010) for M(p, ω) term.

In summary, The final radiance at position p received by eyesis $E_{o}(p)+E_{t}(p)+E_{e}(p)$.

Results and Discussion

We test our method using some three-dimensional models of corn canopy. Fig. 2 shows the simulation results using our algorithm with different parameters in the diffuse and transmittance terms of our appearance model. It is interesting that when we set all the pigment content parameters equals 0, an albino maize is naturally simulated shown in Fig. 2(f) which consists of the actual cause of the albinism: pigments deficiency.



Fig. 2. The appearance of corn leaves with different parameters of our appearance model. (a) $C_{ab} = 40$, $C_r = 5$, $C_b = 0.0$, N = 1.5; (b) $C_{ab} = 25$, $C_r = 5$, $C_b = 0.0$, N = 1.5; (c) $C_{ab} = 10$, $C_r = 20$, $C_b = 0.0$, N = 1.5; (d) $C_{ab} = 0$, $C_r = 5$, $C_b = 0.0$, N = 1.5; (f) $C_{ab} = 0$. $C_r = 5$, $C_b = 0.2$ and N = 1.5.

Appearance textures is a valid way to model spatial-varying optical property in our method. We prove it by simulating some physiological disease which could form complex textures using pigment textures. Fig. 3 shows the visual result of simulating the appearance responses of corn leaves to nitrogen deficiency, magnesium deficiency compared with the true photograph.

The contribution of Illumination computation to the final visualization is as important as appearance model. Fig. 4 shows our results with changing of lighting conditions. In order to show our illumination calculation more clearly, the results of sky lighting only, sun lighting only and the combined lighting are given, respectively.



Fig. 3. The appearance responses of corn leaves to nitrogen deficiency, magnesium deficiency.



Fig. 4. Simulation results with changing of lighting conditions.

We measured the running speed of our approach by rendering a model has about 8605600 triangles. The FPS (frames per second) is 31 which can prove that the advantage of our method on real-time interaction.

To demonstrate the robustness of our model, we compared the results of our appearance model with the actual measurement data (Fig. 6, Tables 1 and 2). We estimated the RGB value of diffuse reflectance and transmission reflectance of actual corn leaf sample by analysis of multiple images with different light directions (Miao *et al.* 2016). The leaf sample with a circular shape was made by punch bear. The average RGB value of diffuse and transmission of all the points on the sample was used as the diffuse and transmission reflectance of this sample. The values of C_{ab} , C_r ,

and C_m of the leaf sample was accurately measured by actual biochemical experiment. Chlorophyll a+b(C_{ab}), and total carotenoids (C_r) were extracted in acetone 80%. Their concentration was determined using spectrophotometer. Dry matter content C_m is the weight of leaf samples without chlorophyll a+b and carotenoids were. Like (Jacquemoud 1990), we also set N equals 1.5 for maize leaves and set C_b equals 0.0. We used these values as input parameters of our appearance model, and generated the simulation results of diffuse and transmission reflectance. The average differences of normalized pixel values of RGB channels between simulated diffuse and real diffuse are, respectively 0.022, 0.011 and 0.018. The average differences of RGB channels of transmission are, respectively 0.021, 0.011 and 0.019.



Fig. 5. Comparison of simulation results and real data.

Table 1. Simulated results and real data of corn leaf samples.	Table	1. Simulated	results and	real data of	corn	leaf samples.
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Sample number	Simulated diffuse	Simulated transmission	Real diffuse	Real transmission
1	0.0437,0.1252 0.0436	0.0632 0.2875 0.0095	0.06270.11370.000	0.0106 0.19610.0156
2	0.0382 0.0788 0.0435	0.0159 0.1990 0.0031	0.07450.12550.0078	0.06270.13730.0039
3	0.3933 0.3041 0.0540	0.6067 0.5378 0.0808	0.41170.34900.0510	0.52550.4863 0.0352
4	0.0940 0.2329 0.0578	0.2034 0.4464 0.0942	0.10590.19610.0311	0.16860.29800.0274

Table 2. Differences of simulated results and real data of samples.

Sample number	Diffuse Error(R\G\B)	Transmission Error(R\G\B)
1	0.019, 0.015 0.0436	0.0526 0.0914 0.0061
2	0.0363 0.0467 0.0357	0.0468 0.0617 0.0008
3	0.0184 0.0450 0.0030	0.0812 0.0515 0.0456
4	0.0119 0.0368 0.0267	0.0348 0.1484 0.0668

Because some parameters of our approach are usually involved in agriculture and other related areas such as biology, botany, we believe it has greater potential as a visualization tool for these disciplines than other 3D modeling and visualization methods for the appearance of crop leaves.

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